# SADLER MATHEMATICS METHODS UNIT 2

## WORKED SOLUTIONS

### Chapter 4 Series

### Exercise 4A

**Question 1**

$a = 8, d = 6$

**a** $$\begin{aligned} S_4 &= \frac{4}{2}[2(8) + 3(6)] \\ &= 68 \end{aligned}$$

**b** $$\begin{aligned} S_5 &= \frac{5}{2}[2(8) + 4(6)] \\ &= 100 \end{aligned}$$

**c** $$\begin{aligned} S_6 &= \frac{6}{2}[2(8) + 5(6)] \\ &= 138 \end{aligned}$$

**Question 2**

$a = 28, d = 3$

**a** $$\begin{aligned} S_2 &= \frac{2}{2}[2(28) + 1(-3)] \\ &= 53 \end{aligned}$$

**b** $$\begin{aligned} S_6 &= \frac{6}{2}[2(28) + 5(-3)] \\ &= 123 \end{aligned}$$

**c** $S_1 = a = 28$

## Question 3

$a = -6, d = 3$

**a** $S_2 = \frac{2}{2}[2(-6) + 1(3)]$
$= -9$

**b** $S_5 = \frac{5}{2}[2(-6) + 4(3)]$
$= 0$

**c** $S_6 = \frac{6}{2}[2(-6) + 5(3)]$
$= 9$

## Question 4

$T_1 = 5(1) + 1 = 6$
$T_2 = 5(2) + 1 = 11$
$T_3 = 5(3) + 1 = 16$
$T_4 = 5(4) + 1 = 21$

$S_1 = 6$
$S_2 = 6 + 11 = 17$
$S_3 = 17 + 16 = 33$
$S_4 = 33 + 21 = 54$

## Question 5

$T_1 = 11$
$T_2 = 11 + 3 = 14$
$T_3 = 14 + 3 = 17$
$T_4 = 17 + 3 = 20$

$S_1 = 11$
$S_2 = 11 + 14 = 25$
$S_3 = 25 + 17 = 42$
$S_4 = 42 + 20 = 62$

**Question 6**

$T_1 = 25 - 3(1) = 22$
$T_2 = 25 - 3(2) = 19$
$T_3 = 25 - 3(3) = 16$
$T_4 = 525 - 3(4) = 13$

$S_1 = 22$
$S_2 = 22 + 19 = 41$
$S_3 = 41 + 16 = 57$
$S_4 = 57 + 13 = 70$

**Question 7**

$$\begin{aligned}
T_1 &= S_1 = 25 \\
T_2 &= S_2 - S_1 \\
&= 57 - 25 \\
&= 32 \\
T_3 &= S_3 - S_2 \\
&= 96 - 57 \\
&= 39 \\
T_4 &= S_4 - S_3 \\
&= 142 - 96 \\
&= 46 \\
T_5 &= S_5 - S_4 \\
&= 195 - 142 \\
&= 53
\end{aligned}$$

25, 32, 39, 46, 53 is an arithmetic sequence

## Question 8

$$
\begin{aligned}
T_1 &= S_1 = 1 \\
T_2 &= S_2 - S_1 \\
&= 5 - 1 \\
&= 4 \\
T_3 &= S_3 - S_2 \\
&= 14 - 5 \\
&= 9 \\
T_4 &= S_4 - S_3 \\
&= 30 - 14 \\
&= 16 \\
T_5 &= S_5 - S_4 \\
&= 55 - 30 \\
&= 25
\end{aligned}
$$

1,4 ,9, 16, 25 is not an arithmetic sequence

## Question 9

$a = 5, d = 11$

**a** $$\begin{aligned} S_3 &= \frac{3}{2}[2(5) + 2(11)] \\ &= 48 \end{aligned}$$

**b** $$\begin{aligned} S_{40} &= \frac{40}{2}[2(5) + 39(11)] \\ &= 8780 \end{aligned}$$

## Question 10

$a = 60, d = -2$

**a** $S_3 = \frac{3}{2}[2(60) + 2(-2)]$

$= 174$

**b** $S_{60} = \frac{60}{2}[2(60) + 59(-2)]$

$= 60$

## Question 11

$a = 1, l = 100, n = 100$

$S_{100} = \frac{100}{2}[1 + 100]$

$= 5050$

## Question 12

$a = 16, d = 4$

**a** $T_{29} = 16 + 27(4)$

$= 128$

or

$T_n = a + (n-1)d$

$128 = 16 + (n-1)4$

$4(n-1) = 11$

$n - 1 = 28$

$n = 29$

**b** $S_{29} = \frac{29}{2}[2(16) + 28(4)]$

$= 2088$

## Question 13

$a = 9, d = 17$

**a** $$\begin{aligned} T_n &= a + (n-1)d \\ 689 &= 9 + (n-1)7 \\ 17(n-1) &= 680 \\ n-1 &= 40 \\ n &= 41 \end{aligned}$$

**b** $$\begin{aligned} S_{41} &= \frac{41}{2}[2(9) + 40(17)] \\ &= 14\,309 \end{aligned}$$

## Question 14

$20, 22, 24... \Rightarrow a = 2, d = 2$

**a** $$\begin{aligned} T_{30} &= 20 + 29(2) \\ &= 78 \text{ km} \end{aligned}$$

**b** $$\begin{aligned} S_{30} &= \frac{30}{2}[2(20) + 29(2)] \\ &= 1470 \text{ km} \end{aligned}$$

## Question 15

$5, 7, 9... \Rightarrow a = 5, d = 2$

$$\begin{aligned} S_{15} &= \frac{15}{2}[2(5) + 14(2)] \\ &= 285 \text{ trees} \end{aligned}$$

## Question 16

$4000, 3750, 3500... \Rightarrow a = 4000, d = -250$

$$\begin{aligned} S_{12} &= \frac{12}{2}[2(4000) + 11(-250)] \\ &= \$31\,500 \end{aligned}$$

**Question 17**

**a** $65\,000, 67\,500, 70\,000 ... \Rightarrow a = 65\,000, d = 2500$

$$S_{10} = \frac{10}{2}[2(65\,000) + 9(2500)]$$
$$= \$762\,500$$

**b**

$68\,000, 69\,200, 70\,400 ... \Rightarrow a = 68\,000, d = 1200$

$$S_{10} = \frac{10}{2}[2(68\,000) + 9(1200)]$$
$$= \$734\,000$$

**Question 18**

| Month | Balance | Interest |
|---|---|---|
| 1 | 36 000 | 720 |
| 2 | 34 000 | 680 |
| 3 | 32 000 | 640 |

Interest payments form an arithmetic sequence with $a = 720, d = -40$

$$S_{18} = \frac{18}{2}[2(720) + 17(-40)]$$
$$= \$6840$$

# Exercise 4B

## Question 1

$T_1 = 2(3)^1 = 6$

$T_2 = 2(3)^2 = 18$

$T_3 = 2(3)^3 = 54$

$T_4 = 2(3)^4 = 162$

$$\begin{aligned}
S_1 &= T_1 = 6 \\
S_2 &= T_1 + T_2 \\
&= 6 + 18 \\
&= 24 \\
S_3 &= S_2 + T_3 \\
&= 24 + 54 \\
&= 78 \\
S_4 &= S_3 + T_4 \\
&= 78 + 162 \\
&= 240
\end{aligned}$$

**Question 2**

$$
\begin{aligned}
T_2 &= 1.5T_1 \\
24 &= 1.5T_1 \\
T_1 &= 16 \\
T_3 &= 1.5T_2 \\
&= 1.5 \times 24 \\
&= 36 \\
T_4 &= 1.5T_3 \\
&= 1.5 \times 36 \\
&= 54
\end{aligned}
$$

$$
\begin{aligned}
S_1 &= T_1 = 16 \\
S_2 &= T_1 + T_2 \\
&= 16 + 24 \\
&= 40 \\
S_3 &= S_2 + T_3 \\
&= 40 + 36 \\
&= 76 \\
S_4 &= S_3 + T_4 \\
&= 76 + 54 \\
&= 130
\end{aligned}
$$

## Question 3

$$
\begin{aligned}
T_1 &= S_1 = 1\\
T_2 &= S_2 - S_1\\
&= 2 - 1\\
&= 1\\
T_3 &= S_3 - S_2\\
&= 4 - 2\\
&= 2\\
T_4 &= S_4 - S_3\\
&= 7 - 4\\
&= 3\\
T_5 &= S_5 - S_4\\
&= 12 - 7\\
&= 5
\end{aligned}
$$

1, 1, 2, 3, 5 is not a geometric sequence

## Question 4

$$
\begin{aligned}
T_1 &= S_1 = 8\\
T_2 &= S_2 - S_1\\
&= 32 - 8\\
&= 24\\
T_3 &= S_3 - S_2\\
&= 104 - 32\\
&= 72\\
T_4 &= S_4 - S_3\\
&= 320 - 104\\
&= 216\\
T_5 &= S_5 - S_4\\
&= 968 - 320\\
&= 648
\end{aligned}
$$

8, 24, 72, 216, 648 is a geometric sequence

**Question 5**

$$a = 1, r = 1$$

$$\begin{aligned} S_{15} &= \frac{1(2^{15} - 1)}{2 - 1} \\ &= 32767 \end{aligned}$$

**Question 6**

$$a = 20\ 480, r = 0.5$$

$$\begin{aligned} S_{15} &= \frac{20\ 480(1 - 0.5^{11})}{1 - 0.5} \\ &= 40\ 940 \end{aligned}$$

**Question 7**

$$\begin{aligned} S_9 &= \frac{256(2.5^9 - 1)}{2.5 - 1} \\ &= 650\,871 \end{aligned}$$

**Question 8**

$$\begin{aligned} S_9 &= \frac{62\ 500(1 - 0.4^9)}{1 - 0.4} \\ &= 104\ 139.36 \end{aligned}$$

## Question 9

$a = 2.25, r = 4$

$$S_6 = \frac{2.25(4^6 - 1)}{4 - 1}$$
$$= 3071.25$$

$$S_7 = 3071.25 + 9216$$
$$= 12\ 287.25$$

$$T_8 = 9216 \times 4$$
$$= 36\ 684$$
$$S_8 = 12\ 287.25 + 36\ 864$$
$$= 49\ 151.25$$

## Question 10

$a = 5, r = 2$

$$S_n = \frac{5(2^n - 1)}{2 - 1} > 5\ 000\ 000$$

Solve by classpad or

$$5(2^n - 1) > 5\ 000\ 000$$
$$2^n - 1 > 1\ 000\ 000$$
$$2^n > 1\ 000\ 001$$
$$n > 19.93$$
$$\therefore n = 20$$

**Question 11**

$a = 28, r = 1.5$

$S_n = \dfrac{28(1.5^n - 1)}{1.5 - 1} > 1\,000\,000$

By classpad,

$n > 24.1$

$\therefore n = 25$

**Question 12**

$T_3 = ar^2 = 24$

$T_4 = ar^3 = 96$

$\Rightarrow r = 4$

$T_3 = a(4)^2 = 24$

$a = 1.5$

$$\begin{aligned} T_{10} &= 1.5(4)^9 \\ &= 393\,216 \end{aligned}$$

$$\begin{aligned} S_{10} &= \frac{1.5(4^{10} - 1)}{4 - 1} \\ &= 524\,287.5 \end{aligned}$$

**Question 13**

$50\ 000, 57\ 500, 66\ \ 125...\qquad a = 50\ 000, r = 1.15$

$$\begin{aligned} S_{10} &= \frac{50\ 000(1.15^{10} - 1)}{1.15 - 1} \\ &= 1\,015\,185.91 \end{aligned}$$

In 10 years, $1 1015 000

## Question 14

$a = 5000, r = 1.1$

**a** $5000 \times 1.1 = 5500$ tonnes

**b** $5500 \times 1.1 = 6050$ tonnes

**c** $6050 \times 1.1 = 6655$ tonnes

**d** $$S_{12} = \frac{5000(1.1^{12} - 1)}{1.1 - 1}$$
$$= 106\ 921.4$$

107 000 tonnes

## Question 15

**a** $60\ 000 \times 1.15 = \$69\ 000$

**b** $69\ 000 \times 1.15 = \$79\ 350$

**c** $$T_{10} = 60\ 000(1.15)^9$$
$$= 211\ 072.58$$
$$\therefore \$211\ 000$$

**d** $$S_{10} = \frac{60\ 000(1.15^{10} - 1)}{1.15 - 1}$$
$$= \$1\ 218\ 223.10$$
$$\therefore \$1\ 218\ 000$$

**Question 16**

On 1/1/18 $1200\times1.1^4+1200\times1.1^3+1200\times1.1^2+1200\times1.1+1200$

On 1/1/29 $1200\times1.1^{15}+1200\times1.1^{14}+1200\times1.1^{13}+...+1200$

$$=1200(1+1.1+1.1^2+...+1.1^{15})$$

$$=1200\times1\frac{(1.1^{16}-1)}{0.1}$$

$$=43\,139.68$$

$$=43\,140$$

\$43 140 in the account

**Question 17**

Dec 2015 $1000\times1.07$

Dec 2016 $1000\times1.07^2+1000\times1.07$

Dec 2017 $1000\times1.07^3+1000\times1.07^2+1000\times1.07$

$\vdots$

Dec 2024 $1000\times1.07^{10}+1000\times1.07^9+...+1000\times1.07$

$$S_{10}=\frac{1070(1.07^{10}-1)}{1.07-1}$$

$$=14\ 783.60$$

$\therefore$ \$14 784 to the nearest dollar

## Question 18

$2500,\ 2550,\ 2601 \Rightarrow a = 2500, r = 1.02$

**a** $T_4 = 2500 \times 1.03^3$
$= 2653$

**b** $T_{15} = 2500 \times 1.02^{14}$
$= 3299$

**c** $T_{16} = 3299$

**d** $S_{15} = \dfrac{2500(1.02^{15} - 1)}{1.02 - 1}$
$= 43\ 233$
$\therefore 43\ 200$ items to the nearest hundred

**e** $43\ 233 + 25 \times 3299$
$= 125\ 708$
$\therefore 125\ 700$ items to the nearest hundred

## Question 19

8000, 8000, 8000, 7200, 6480...

**a** $8000 \times 0.9^{n-1} = 1900$
$n - 1 = 13.6$
$n = 14.6$
$\therefore$ during year 17

**b** $S_{15} = \dfrac{8000(1 - 0.9^{15})}{1 - 0.9}$
$= 63528.7$

Total: $63\ 528.7 + 16\ 000$
$= 79\ 528.7$
$\approx 79500$ tonnes

**Question 20**

**a** $a = P, r = 1.095, n = 21$

**b** $S_{21} = \dfrac{P(1.095^{21} - 1)}{1.095 - 1}$

$= 50\ 000$

By classpad, $P = \$829.7$

**Question 1**

Geometric progression A

$$a = 24, r = \frac{9.6}{24} = 0.4$$

$S_\infty$ exists as $|r| < 1$

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{24}{1-0.4} \\ &= 40 \end{aligned}$$

Geometric progression B

$$a = 8, r = \frac{12}{8} = 1.5$$

$S_\infty$ does not exist as $|r| > 1$

Geometric progression C

$$a = 35, r = \frac{10.5}{35} = 0.3$$

$S_\infty$ exists as $|r| < 1$

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{35}{1-0.3} \\ &= 50 \end{aligned}$$

**Question 2**

**a** $a = 100, r = 0.5 \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{100}{1-0.5} \\ &= 200 \end{aligned}$$

**b** $a = 100, r = 0.75 \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{100}{1-0.75} \\ &= 400 \end{aligned}$$

**c** $a = 100, r = 1.1 \therefore S_\infty$ does not exist

**d** $a = 90, r = 0.8 \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{90}{1-0.8} \\ &= 450 \end{aligned}$$

**e** $a = 56, r = 1.25 \therefore S_\infty$ does not exist

**f** $a = 90, r = -0.8 \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{90}{1-(-0.8)} \\ &= 50 \end{aligned}$$

**g** $a = 0.6, r = \frac{1}{3} \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{0.6}{1-\frac{1}{3}} \\ &= 0.9 \end{aligned}$$

**h** $a = 2304, r = -\frac{1}{8} \therefore S_\infty$ exists

$$\begin{aligned} S_\infty &= \frac{2304}{1-(-\frac{1}{8})} \\ &= 2048 \end{aligned}$$

**Question 3**

$$S_\infty = \frac{a}{1-r}$$
$$120 = \frac{48}{1-r}$$
$$1-r = \frac{48}{120}$$
$$r = \frac{3}{5}$$

**Question 4**

$$S_\infty = \frac{a}{1-r}$$
$$120 = \frac{a}{1-0.45}$$
$$a = 120 \times 0.55$$
$$r = 66$$

## Question 5

| Injection | Before | After | Before | After |
|---|---|---|---|---|
| 1 | 0 | 15 | 0 | 15 |
| 2 | 6 | 21 | $15\times0.4$ | $15\times0.4+15$ |
| 3 | 8.4 | 23.4 | $15\times0.4^2+15\times0.4$ | $15\times0.4^2+15\times0.4+15$ |
| 4 | 9.36 | 24.36 | $15\times0.4^3+15\times0.4^2+15\times0.4$ | $15\times0.4^3+15\times0.4^2+15\times0.4+15$ |

**a** The terms in the 'After' column can be written as $15(1+0.4+0.4^2+0.4^3+...)$

$$15\times S_\infty$$
$$=15\times\frac{1}{1-0.4}$$
$$=25\text{ mg}$$

**b** The terms in the 'After' column can be written as $15(0.4+0.4^2+0.4^3+...)$

$$15\times S_\infty$$
$$=15\times\frac{0.4}{1-0.4}$$
$$=10\text{ mg}$$

## Question 6

$50, 40, 32 \Rightarrow a=50, r=0.8$

$$S_\infty=\frac{50}{1-0.2}$$
$$=250$$

See the paragraph in text for a comment.

## Question 7

**a** $60\% \times 2 = 1.2 \text{ m}$

**b** $2 \times 0.6^6 = 0.0933 \text{ m}$

$= 9.3 \text{ cm}$

Approximately 9 cm.

**c** Consider the drop heights only: $2, 1.2, 0.72....$

$$S_\infty = \frac{2}{1-0.6}$$
$$= 5 \text{ m}$$

The total distance travelled is $5 \times 2 - 2 = 8 \text{ m}$

(The first 2 m does not count as the ball does not bounce up to start.)

## Question 8

5, 2, 0.8, 0.32... $a = 5, r = 0.4$

$$S_\infty = \frac{5}{1-0.4}$$
$$= 8\tfrac{1}{3} \text{ m}$$

The total distance travelled is $8\tfrac{1}{3} \times 2 - 5 = 11\tfrac{2}{3} \text{ m}$.

**Question 1**

**a** $2^6$

**b** $2^8$

**c** $2^7$

**d** $2^2$

**e** $2^{10}$

**f** $2^2$

**g** $2^2 \times 2^3 \times 2^4 \times 2^5 = 2^{14}$

**h** $2^0$

**i**

$$\begin{aligned} 36-4 &= 32 \\ 32 &= 2^5 \end{aligned}$$

**Question 2**

$$\frac{1}{2}$$

**Question 3**

$$4^2 = 16$$

**Question 4**

$$\frac{9}{4} = 2\tfrac{1}{4}$$

**Question 5**

1

**Question 6**

$4^3 = 64$

**Question 7**

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

**Question 8**

$$\frac{3^7 \times (3^3)^2}{3^{14}}$$
$$= \frac{3^7 \times 3^6}{3^{14}}$$
$$= \frac{1}{3}$$

**Question 9**

$$\frac{5^8}{5^4} \times \frac{1}{5^3}$$
$$= 5$$

## Question 10

$$\frac{7^{10}}{7^2 \times (7^2)^2 \times 7^5}$$
$$= \frac{7^{10}}{7^{11}}$$
$$= \frac{1}{7}$$

## Question 11

$$\frac{2^n}{2^n} = 1$$
$$\frac{2^n}{2^n} = 2^{n-n} = 2^0$$
$$\therefore 2^0 = 1$$

## Question 12

**a** 17, 22, 27, 32, 37

**b** 100, 93, 86, 79, 72

**c** 5, 15, 45, 135, 405

**d** $T_1 = 6, T_{n+1} = T_n + 4$

**e** $T_1 = 2, T_{n+1} = 3T_n$

**f** $T_1 = 17, T_{n+1} = T_n - 8$

**Question 13**

**a** Exponential pattern $\Rightarrow \mathrm{T}_{n+1} = kT_n$

$$T_1 = 1, T_2 = 2, T_3 = 4, T_4 = 8, T_5 = 16$$
$$k = 2$$

$$T_1 = 1, r = 2$$
$$\begin{aligned} T_{20} &= 1 \times 2^{19} \\ &= 524\,288 \end{aligned}$$

**b** Linear pattern $\Rightarrow \mathrm{T}_{n+1} = T_n + a$

$$T_1 = 5,\ T_2 = 8,\ T_3 = 11,\ T_4 = 14,\ T_5 = 17$$
$$T_1 = 5, a = 3$$

$$\begin{aligned} T_{20} &= 5 + 19 \times 3 \\ &= 62 \end{aligned}$$

**Question 14**

**a**
$$\begin{aligned} {}^{6}C_4 &= \frac{6!}{2!4!} \\ &= \frac{6 \times 5 \times 4!}{2 \times 4!} \\ &= 15 \end{aligned}$$

**b**
$$\begin{aligned} {}^{n}C_2 &= \frac{n!}{(n-2)!2!} \\ &= \frac{n(n-1)(n-2)!}{(n-2)!2!} \\ &= \frac{n(n-1)}{2} \end{aligned}$$

### Question 15

**a**

$$8,\ x,\ 50$$

$$50 - x = x - 8$$

$$58 = 2x$$

$$x = 29$$

$$T_1 = 8,\ T_{n+1} = T_n + 21$$

**b**

$$8,\ x,\ 50$$

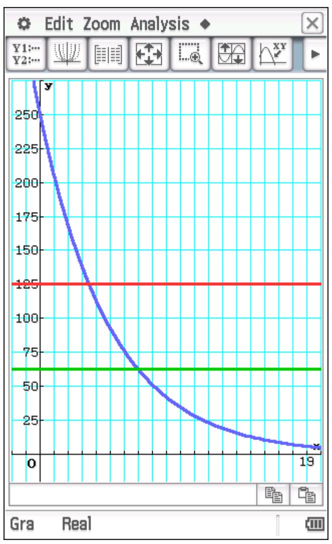
$$\frac{x}{8} = \frac{50}{x}$$

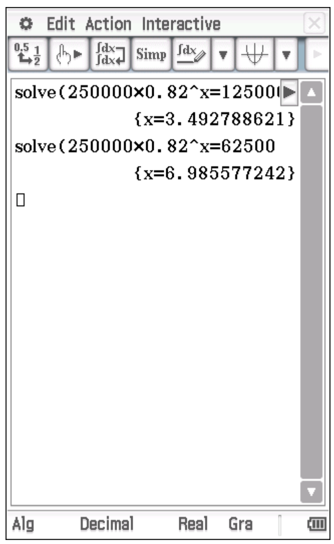
$$x^2 = 400$$

$$x = \pm 20$$

$$T_1 = 8, T_{n+1} = \pm 2.5T_n$$

### Question 16




**a** in approximately 3.5 years time

**b** in approximately 7 years time

**Question 17**

| Day | 1 | 2 | 3 | 4 | 5 | 21 |
|---|---|---|---|---|---|---|
| Days until competition | 21 | 20 | 19 | 18 | 17 | |
| Time | 30 | 33 | 36 | 39 | 42 | |

$$T_n = T_{n+1} + 3, \quad T_1 = 30$$

$$\begin{aligned} T_{21} &= a + 20d \\ &= 30 + 20(3) \\ &= 90 \end{aligned}$$

33, 36, 39... 90

$$\begin{aligned} S_n &= \frac{20}{2}(33 + 90) \\ &= 1230 \text{ minutes} \\ &= 20 \text{ hours } 30 \text{ minutes} \end{aligned}$$

**Question 18**

Option A

Value after 20 years: $1\,000\,000 \times 1.06^{20} = \$3\,207\,135.47$

Option B

Beginning of Y1: $50\,000$

Beginning of Y2: $50\,000 \times 1.06 + 50\,000$

Beginning of Y3: $50\,000 \times 1.06^2 + 50\,000 \times 1.06 + 50\,000$

End of Y20: $50\,000 \times 1.06^{20} + 50\,000 \times 1.06^{19} + \ldots.\ 50\,000 \times 1.06$
$= 50\,000(1.06^{20} + 1.06^{19} + \ldots + 1.06)$

Value after 20 years: $50\,000 \times \dfrac{1.06(1.06^{20} - 1)}{1.06} = 1\,949\,636.334$

Let $x$ represent the amount of money to be invested in the bank account to cover future payments.

At the beginning of year 2, the amount of money in the account is $x \times 1.06 - 50\,000$

At the beginning of year 3, the amount of money in the account is $(x \times 1.06 - 50\,000) \times 1.06 - 50\,000$
$= 1.06^2 x - 50\,000 \times 1.06 - 50\,000$

At the beginning of year 4, the amount of money in the account is
$(1.06^2 x - 50\,000 \times 1.06 - 50\,000) \times 1.06 - 50\,000$
$= 1.06^3 x - 50\,000 \times 1.06^2 - 50\,000 \times 1.06 - 50\,000$

In the last year, year 20, the amount of money in the account is

$1.06^{19} x - 50\,000 \times 1.06^{18} - 50\,000 \times 1.06^{17} - \ldots - 50\,000$
$= 1.06^{19} x - 50\,000(1.06^{18} + 1.06^{17} + \ldots + 1)$

$$1.06^{18} + 1.06^{17} + \ldots + 1 = \frac{1(1.06^{19} - 1)}{0.06}$$

At the end of year 20 the balance of the account is zero.

Using classpad to solve $x \times 1.06^{19} - \dfrac{50\,000 \times (1.06^{19} - 1)}{0.06} = 0$ gives $x = 557\,905.82$

The organisers need to have \$607 906 (nearest dollar) available to cover the first payment of \$50 000 and the investment amount.